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成层竖向排水井地基固结分析

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摘 要: 实际工程中竖井地基具有成层性, 有时竖井也并未完全打穿软土层。在竖井打设区满足竖井等应变固结理论、下卧层满足一维太沙基固结理论假设的基础上, 将现有竖井地基固结理论推广到成层未打穿竖井地基情况。利用边界条件和竖向连续条件, 确定该系统的正交关系, 并给出了其固结解答, 该解具有广泛的适用性。通过对竖井打设区和下卧层层数的变化, 即可获得现有关于简单未打穿竖井地基的固结解答。将该解答编制成应用程序, 对一算例进行了分析。结果表明, 平均固结度按孔压定义和按变形定义是不相同的, 硬表层的存在会加快其下土层的固结。

关 键 词: 成层地基; 竖向排水井; 固结; 硬表层

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Analysis of consolidation of layered ground with vertical drain

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Abstract: In engineering practice, the subsoil with vertical drains is often composed of multi-layer strata and the subsoil is partially penetrated by vertical drain wells. Based on the assumptions that the section with vertical drains satisfies equal strain condition and the section without vertical drains satisfies Terzaghi's one dimensional consolidation theory, the existing simplified consolidation theory of ground with partially penetrated vertical drain wells is developed so that multi-layer subsoil can be take into account. The orthogonal relation of the system is obtained using the boundary conditions and continuity conditions in vertical direction. The consolidation solutions of the system is presented. The solution has comprehensive applicability. The existing consolidation solutions of simplified ground with partially penetrated vertical drain wells can be obtained by changing the number of the stratum of the section with vertical drain wells and the section without it. The solution has been coded into a program and a case is analyzed using it. Analysis shows that the average degree of consolidation of the system defined by pore pressure is different from that defined by settlement. It also shows that existence of crust can accelerate the consolidation of the stratum.

Key words: layered ground; vertical drain wells; consolidation; crust

1 前 言

用竖向排水井加速土体固结的排水固结预压法是一种较为经济有效的地基处理方法。自 Barron^[1]于 1948 年展开对竖井地基的固结解析研究以来, 竖井地基的解析研究已经取得了较大的进展。但现有研究成果大多是针对单层竖井地基而言, 实际土层具有成层性。另外, 由于各种原因竖井未能打穿软土层的现象也时有发生。针对以上工程实际问题, 一些学者分别就成层竖井地基和未打穿竖井地

基的固结问题展开了研究。Horne^[2]最早对成层竖井地基的固结问题进行研究; 赵维炳将改进的半解析解法应用于成层竖井地基的位移分析中^[3]; 谢康和^[4]提出了等应变条件下双层理想井地基的固结理论, 房莹光采用 Laplace-Fourier 变换和传播矩阵技术给出了成层竖井地基固结变形解^[5], Tang^[6]首次给出了考虑径、竖向组合作用以及涂抹、井阻作用下双层竖井地基固结解答。Hart^[7]最先提出未打穿竖井地基的近似计算方法; Zeng^[8]对其近似方法作了改进; Tang^[9]对渗透面上的连续条件作了简化, 给出

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了考虑竖井打设区和下卧层相互影响的未打穿竖井地基固结解。本文对成层地基中打设竖井的一般情况进行了研究,给出了其固结解,并用该解答分析了一工程算例。

2 数学模型

2.1 计算简图

一般成层未打穿竖向排水井地基在瞬时荷载作用下的固结模型如图1所示。图中, r_w 为竖井半径; r_s 为涂抹区半径; r_e 为竖井影响区半径。沿用Hart^[7], Tang^[9]以及Runesson^[10]的定义,将穿过竖井底部的水平面称为渗透面,渗透面以上称为竖井打设区,渗透面以下称为下卧层。 $\perp$ 为下卧层标志;下标 i, j 为土层编号; h_i 为竖井打设区第 i 土层的厚度; $h_{\perp j}$ 为下卧层第 j 土层的厚度,设竖井打设区共分 n 层,下卧层由 N 层组成, k_{vi} , k_{hi} , k_{si} 分别为竖井打设区第 i 土层的竖直向和水平向渗透系数以及涂抹区的渗透系数, m_{vj} 为该层的体积压缩系数, $k_{v\perp j}$ 为下卧层第 j 土层的竖直向渗透系数, $m_{v\perp j}$ 为该层的体积压缩系数, k_w 为竖井的渗透系数,竖井地基顶部为排水面,下卧层底部为不排水面。底部为排水情况,可参照不排水情况求解, r, z 分别为径向和竖向坐标。荷载瞬时施加,均布荷载大小为 $q=q_0$ 。

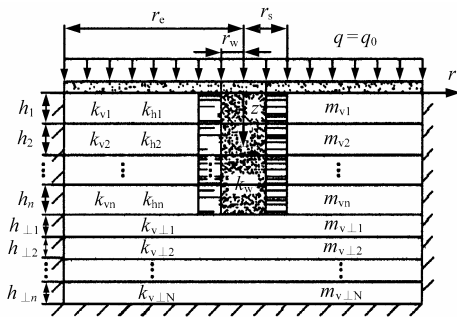


图1 成层未打穿竖向排水井地基计算简图
Fig.1 Scheme of stratified soils with partially penetrated vertical drain

为了叙述问题的方便,令 H_i 为竖井打设区第 i 土层层底坐标, $H_0=0$ 为系统顶面坐标, $H_{\perp j}$ 为下卧层第 j 土层层底坐标, H_t 为土层总厚度。

由于在土层交界面处的固结的复杂性,本文作如下简化:①各土层除了在交界面处满足孔压、流量连续条件外,对其它土层的固结不再产生影响,所有土层均满足等应变假定。②下卧层仅发生竖向固结,水平向固结可忽略不计。根据 Tang^[6,9]竖井打设区准等应变假设,下卧层满足一维太沙基固结理论以及问题本身的特点,可得求解系统应满足

如下控制方程,边界条件,连续条件以及初始条件。

2.2 控制方程

$$\frac{C_{hi}}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_{ni}}{\partial r} \right) + C_{vi} \frac{\partial^2 \bar{u}_i}{\partial z^2} = \frac{\partial \bar{u}_i}{\partial t}, \quad r_s \leq r \leq r_e \quad (1)$$

$$\frac{C_{si}}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u_{si}}{\partial r} \right) + C_{vi} \frac{\partial^2 \bar{u}_i}{\partial z^2} = \frac{\partial \bar{u}_i}{\partial t}, \quad r_w \leq r \leq r_s \quad (2)$$

$$\bar{u}_i = \frac{2\pi}{\pi(r_e^2 - r_w^2)} \left(\int_{r_w}^{r_s} r u_{si} dr + \int_{r_s}^{r_e} r u_{ni} dr \right) \quad (3)$$

$$2\pi r_w \frac{k_{si}}{\gamma_w} \left(\frac{\partial u_{si}}{\partial r} \right) \Big|_{r=r_w} = -\pi r_w^2 \frac{k_w}{\gamma_w} \frac{\partial^2 u_{wi}}{\partial z^2} \quad (4)$$

式中 $i=1, 2, \dots, n$ 为竖井打设区土层序号; $u_{ni}(r, z, t)$, $u_{si}(r, z, t)$, $u_{wi}(z, t)$ 分别为 i 土层的自然区超静孔压,涂抹区超静孔压和竖井区超静孔压; $\bar{u}_i(z, t)$ 为 i 土层的平均孔压; $C_{hi}=k_{hi}/(m_{vi}\gamma_w)$, $C_{vi}=k_{vi}/(m_{vi}\gamma_w)$, $C_{si}=k_{si}/(m_{vi}\gamma_w)$ 。

根据一维太沙基固结理论,可得下卧层各层土体固结满足如下控制方程:

$$C_{v\perp j} \frac{\partial^2 u_{\perp j}}{\partial z^2} = \frac{\partial u_{\perp j}}{\partial t} \quad (5)$$

式中 $j=1, 2, \dots, n$ 为下卧层土层序号; $u_{\perp j}(z, t)$ 为下卧层第 j 层的孔压; $C_{v\perp j}=k_{v\perp j}/(m_{v\perp j}\gamma_w)$ 。

2.3 连续条件

竖井打设区内各土层径向连续条件为

$$\left. \begin{aligned} u_{si} \Big|_{r=r_w} &= u_{wi} \Big|_{r=r_w}; \quad u_{si} \Big|_{r=r_s} = u_{ni} \Big|_{r=r_s}; \\ k_{si} \frac{\partial u_{si}}{\partial r} \Big|_{r=r_s} &= k_{hi} \frac{\partial u_{ni}}{\partial r} \Big|_{r=r_s} \end{aligned} \right\} \quad (6)$$

当竖向连续条件对应于竖井打设区第 i 土层层底 $z=H_i$ 时 ($i=1, 2, \dots, n-1$), 应满足:

$$\left. \begin{aligned} \bar{u}_i &= \bar{u}_{i+1}; \quad u_{wi} = u_{w(i+1)}; \\ k_{vi} \frac{\partial \bar{u}_i}{\partial z} &= k_{v(i+1)} \frac{\partial \bar{u}_{i+1}}{\partial z}; \quad \frac{\partial u_{wi}}{\partial z} = \frac{\partial u_{w(i+1)}}{\partial z} \end{aligned} \right\} \quad (7)$$

对应于下卧层第 j 土层层底, $z=H_{\perp j}$ 时 ($j=1, 2, \dots, N-1$), 应满足:

$$\left. \begin{aligned} u_{\perp j} \Big|_{z=H_{\perp j}} &= u_{\perp(j+1)} \Big|_{z=H_{\perp j}}; \\ k_{v\perp j} \frac{\partial u_{\perp j}}{\partial z} \Big|_{z=H_{\perp j}} &= k_{v\perp(j+1)} \frac{\partial u_{\perp(j+1)}}{\partial z} \Big|_{z=H_{\perp j}} \end{aligned} \right\} \quad (8)$$

渗透面上竖向连续条件为

$$\left. \begin{aligned} u_{wn} &= u_{\perp 1}; \quad \bar{u}_n = u_{\perp 1}; \\ k_{vn} \pi (r_e^2 - r_w^2) \frac{\partial \bar{u}_n}{\partial z} + k_w \pi r_w^2 \frac{\partial u_{wn}}{\partial z} &= k_{v\perp 1} \pi r_e^2 \frac{\partial u_{\perp 1}}{\partial z} \end{aligned} \right\} \quad (9)$$

2.4 边界条件

$$\left. \begin{aligned} & \text{竖井顶面 } z=0 \text{ 时, } u_{w1}=0, \quad \bar{u}_1=0 \\ & \text{竖井打设区侧面 } r=r_e \text{ 时, } \frac{\partial u_{ni}}{\partial r}=0 \\ & \text{下卧层底面 } z=H_t \text{ 时, } \frac{\partial u_{\perp N}}{\partial z}=0 \end{aligned} \right\} \quad (10)$$

2.5 初始条件

$$\left. \begin{aligned} & \text{竖井打设区各土层: } \bar{u}_i(z,0)=q_0, \quad i=1,2,\dots,n \\ & \text{下卧层各土层: } u_{\perp j}(z,0)=q_0, \quad j=1,2,\dots,N \end{aligned} \right\} \quad (11)$$

3 问题求解

采用文献[6]和文献[9]的求解思路, 可以求得竖井打设区和下卧层各土层孔压的通解形式:

$$u_{wi}(z,t)=\sum_{m=0}^{\infty} Z_{wim}(z) T_m(t)=\sum_{m=0}^{\infty} A_m g_{wim}(z) e^{-\beta_m t} \quad (12)$$

$$\bar{u}_i(z,t)=\sum_{m=0}^{\infty} Z_{im}(z) T_m(t)=\sum_{m=0}^{\infty} A_m g_{im}(z) e^{-\beta_m t} \quad (13)$$

$$u_{\perp j}(z,t)=\sum_{m=0}^{\infty} Z_{\perp jm}(z) T_m(t)=\sum_{m=0}^{\infty} A_m g_{\perp jm}(z) e^{-\beta_m t} \quad (14)$$

式中 $g_{wim}(z)$, $g_{im}(z)$, $g_{\perp jm}(z)$ 意义如下:

$$g_{wim}(z)=a_{im} \sin(\lambda_{im} z/H_t)+b_{im} \cos(\lambda_{im} z/H_t)+c_{im} \exp(\xi_{im} z/H_t)+d_{im} \exp(-\xi_{im} z/H_t);$$

$$g_{im}(z)=\left(1+\frac{\lambda_{im}^2}{\varphi_i}\right)\left[a_{im} \sin\left(\frac{\lambda_{im} z}{H_t}\right)+b_{im} \cos\left(\frac{\lambda_{im} z}{H_t}\right)\right]+\left(1-\frac{\xi_{im}^2}{\varphi_i}\right)\left[c_{im} \exp\left(\frac{\xi_{im} z}{H_t}\right)+d_{im} \exp\left(\frac{-\xi_{im} z}{H_t}\right)\right];$$

$$g_{\perp jm}(z)=a_{\perp jm} \sin[\lambda_{\perp jm}(1-z/H_t)]+b_{\perp jm} \cos[\lambda_{\perp jm}(1-z/H_t)].$$

其中, β_m 为待求特征根; A_m , a_{im} , b_{im} , c_{im} , d_{im} , $a_{\perp jm}$, $b_{\perp jm}$ 为待定系数。而

$$\Xi_{im}=-\left\{\frac{2C_{hi}}{r_e^2 F_i}\left[1+\frac{k_{vi}}{k_w}(n^2-1)\right]-\beta_m\right\};$$

$$\varphi_i=2(n^2-1)k_{hi}H_t^2/(F_i k_w r_e^2);$$

$$\Theta_{im}=-2(n^2-1)k_{hi}\beta_m/(r_e^2 F_i k_w);$$

$$\xi_{im}=H_t \sqrt{(-\Xi_{im}+\sqrt{\Xi_{im}^2-4C_{vi}\Theta_{im}})/2C_{vi}};$$

$$\lambda_{im}=H_t \sqrt{(\Xi_{im}+\sqrt{\Xi_{im}^2-4C_{vi}\Theta_{im}})/2C_{vi}};$$

$$F_i=n^2/(n^2-1)[\ln(n/s)+\ln(s)k_{hi}/k_{si}-0.75]+ \frac{s^2}{n^2-1}\left(1-\frac{k_{hi}}{k_{si}}\right)\left(1-\frac{s^2}{4n^2}\right)+\frac{k_{hi}}{k_{si}}\frac{1}{n^2-1}\left(1-\frac{1}{4n^2}\right);$$

$$n=r_e/r_w; \quad s=r_s/r_w; \quad \lambda_{\perp jm}=H_t \sqrt{\beta_m/C_{v\perp j}}.$$

(15)

根据竖向连续条件式(7)~(9)可得:

$$SX=0 \quad (16)$$

式中 $X=[XX_1 \quad XX_2 \cdots XX_{n+N}]^T$; $XX_i=(a_{im} \quad b_{im} \quad c_{im} \quad d_{im})$,

$i=1, 2, \dots, n$; $XX_{n+j}=(a_{\perp jm} \quad b_{\perp jm})$, $j=1, 2, \dots, N$;

$$S=\begin{bmatrix} P^1 & Q^1 & & & \\ & P^2 & Q^2 & & \\ & & \ddots & \ddots & \\ & & & P^{n+N-1} & Q^{n+N-1} \end{bmatrix}, \text{空白处子矩阵均为 } 0$$

矩阵。

当 $i=1, 2, \dots, n-1$ 时, P^i , Q^i 均为 4×4 矩阵,

$$P_{11}^i=\sin(\lambda_{im} H_i/H_t); \quad P_{12}^i=\cos(\lambda_{im} H_i/H_t);$$

$$P_{13}^i=\exp(\xi_{im} H_i/H_t); \quad P_{14}^i=\exp(-\xi_{im} H_i/H_t);$$

$$P_{21}^i=(1+\lambda_{im}^2/\varphi_i)P_{11}^i; \quad P_{22}^i=(1+\lambda_{im}^2/\varphi_i)P_{12}^i;$$

$$P_{23}^i=(1-\xi_{im}^2/\varphi_i)P_{13}^i; \quad P_{24}^i=(1-\xi_{im}^2/\varphi_i)P_{14}^i;$$

$$P_{31}^i=\frac{\lambda_{im}}{H_t}P_{12}^i; \quad P_{32}^i=-\frac{\lambda_{im}}{H_t}P_{11}^i; \quad P_{33}^i=\frac{\xi_{im}}{H_t}P_{13}^i;$$

$$P_{34}^i=-\frac{\xi_{im}}{H_t}P_{14}^i; \quad P_{41}^i=k_{vi}\frac{\lambda_{im}}{H_t}P_{22}^i; \quad P_{42}^i=\frac{k_{vi}\lambda_{im}}{-H_t}P_{21}^i;$$

$$P_{43}^i=P_{23}^i k_{vi} \xi_{im}/H_t; \quad P_{44}^i=-P_{24}^i k_{vi} \xi_{im}/H_t;$$

$$Q_{11}^i=-\sin\left[\lambda_{(i+1)m}\frac{H_i}{H_t}\right]; \quad Q_{12}^i=-\cos\left[\lambda_{(i+1)m}\frac{H_i}{H_t}\right];$$

$$Q_{13}^i=-\exp[\xi_{(i+1)m} H_i/H_t]; \quad Q_{14}^i=-\exp[-\xi_{(i+1)m} H_i/H_t];$$

$$Q_{21}^i=\left(1+\frac{\lambda_{(i+1)m}^2}{\varphi_{i+1}}\right)Q_{11}^i; \quad Q_{22}^i=\left(1+\frac{\lambda_{(i+1)m}^2}{\varphi_{i+1}}\right)Q_{12}^i;$$

$$Q_{23}^i=\left(1-\frac{\xi_{(i+1)m}^2}{\varphi_{i+1}}\right)Q_{13}^i; \quad Q_{24}^i=\left(1-\frac{\xi_{(i+1)m}^2}{\varphi_{i+1}}\right)Q_{14}^i;$$

$$Q_{31}^i=Q_{12}^i \lambda_{(i+1)m}/H_t; \quad Q_{32}^i=-Q_{11}^i \lambda_{(i+1)m}/H_t;$$

$$Q_{33}^i=Q_{13}^i \xi_{(i+1)m}/H_t; \quad Q_{34}^i=-Q_{14}^i \xi_{(i+1)m}/H_t;$$

$$Q_{41}^i=k_{v(i+1)}\frac{\lambda_{(i+1)m}}{H_t}Q_{22}^i; \quad Q_{42}^i=k_{v(i+1)}\frac{\lambda_{(i+1)m}}{-H_t}Q_{21}^i;$$

$$Q_{43}^i=k_{v(i+1)}\frac{\xi_{(i+1)m}}{H_t}Q_{23}^i; \quad Q_{44}^i=-k_{v(i+1)}\frac{\xi_{(i+1)m}}{H_t}Q_{24}^i$$

(17)

当 $i=n$ 时, $\mathbf{P}^n$ 为 3×4 矩阵, $\mathbf{Q}^n$ 为 3×2 矩阵。

$$\left. \begin{aligned} P_{11}^n &= \sin(\lambda_{nm} H_n / H_t); & P_{12}^n &= \cos(\lambda_{nm} H_n / H_t); \\ P_{13}^n &= \exp(\xi_{nm} H_n / H_t); & P_{14}^n &= \exp(-\xi_{nm} H_n / H_t); \\ P_{21}^n &= (1 + \lambda_{nm}^2 / \varphi_n) P_{11}^n; & P_{22}^n &= (1 + \lambda_{nm}^2 / \varphi_n) P_{12}^n; \\ P_{23}^n &= (1 - \xi_{nm}^2 / \varphi_n) P_{13}^n; & P_{24}^n &= (1 - \xi_{nm}^2 / \varphi_n) P_{14}^n; \\ P_{31}^n &= (\lambda_{nm} / H_t) [k_{vn} (n^2 - 1) P_{22}^n + k_w P_{12}^n]; \\ P_{32}^n &= -(\lambda_{nm} / H_t) [k_{vn} (n^2 - 1) P_{21}^n + k_w P_{11}^n]; \\ P_{33}^n &= (\xi_{nm} / H_t) [k_{vn} (n^2 - 1) P_{23}^n + k_w P_{13}^n]; \\ P_{34}^n &= (\xi_{nm} / H_t) [k_{vn} (n^2 - 1) P_{24}^n + k_w P_{14}^n]; \\ Q_{11}^n &= -\sin[\lambda_{\perp 1m} \frac{H_n}{H_t}]; & Q_{12}^n &= -\cos[\lambda_{\perp 1m} \frac{H_n}{H_t}]; \\ Q_{21}^n &= Q_{11}^n; & Q_{22}^n &= Q_{12}^n; & Q_{31}^n &= -Q_{12}^n n^2 k_{v\perp 1} \left(\frac{\lambda_{\perp 1m}}{H_t} \right); \\ Q_{32}^n &= Q_{11}^n n^2 k_{v\perp 1} \left(\frac{\lambda_{\perp 1m}}{H_t} \right) \end{aligned} \right\} \quad (18)$$

当 $i=n+j$, $j=1, 2, \dots, N-1$ 时, $\mathbf{P}^i$ 与 $\mathbf{Q}^i$ 均为 2×2 矩阵,

$$\left. \begin{aligned} P_{11}^{n+j} &= \sin[\lambda_{\perp jm} (1 - H_{\perp jm} / H_t)]; \\ P_{12}^{n+j} &= \cos[\lambda_{\perp jm} (1 - H_{\perp jm} / H_t)]; \\ P_{21}^{n+j} &= \frac{k_{v\perp j} \lambda_{\perp jm}}{-H_t} P_{12}^{n+j}; & P_{22}^{n+j} &= \frac{k_{v\perp j} \lambda_{\perp jm}}{H_t} P_{11}^{n+j}; \\ Q_{11}^{n+j} &= -\sin[\lambda_{\perp(j+1)m} (1 - H_{\perp(j+1)m} / H_t)]; \\ Q_{12}^{n+j} &= -\cos[\lambda_{\perp(j+1)m} (1 - H_{\perp(j+1)m} / H_t)]; \\ Q_{21}^{n+j} &= -Q_{12}^{n+j} k_{v\perp(j+1)} \lambda_{\perp(j+1)m} / H_t; \\ Q_{22}^{n+j} &= Q_{11}^{n+j} k_{v\perp(j+1)} \lambda_{\perp(j+1)m} / H_t \end{aligned} \right\} \quad (19)$$

由以上这些公式可知, $\mathbf{S}$ 矩阵实际行数为 $4n+2N-3$, 实际列数为 $4n+2N$, $\mathbf{X}$ 实际为 $4n+2N$ 维列向量, $\mathbf{P}$, $\mathbf{Q}$ 系列矩阵中各数, 均仅含未知数 β_m 。由边界条件(10)可知, $b_{1m}=0$, $d_{1m}=-c_{1m}$, $a_{\perp Nm}=0$ 。由此, 可将 $\mathbf{S}$ 矩阵中第 3 列减去第 4 列, 再将第 2, 4, $4n+2N$ 三列删除得到 $\mathbf{S}^*$ 矩阵, 将 $\mathbf{X}$ 列向量中第 2, 4, $4n+2N$ 三项删除, 得到 $\mathbf{X}^*$ 列向量, 则下式成立:

$$\mathbf{S}^* \times \mathbf{X}^* = 0 \quad (20)$$

因为 $\mathbf{X}^*$ 非零, 所以 $\mathbf{S}^*$ 行列式应为 0:

$$\det(\mathbf{S}^*) = 0 \quad (21)$$

通过式 (21) 可求得 β_m 。由于 a_{1m} 包含于 A_m 中, 令 $a_{1m}=1$, 通过式 (20) 可得 $\mathbf{X}^*$ 中其它待定未知数。

可以参照文献[6]求双层地基的正交关系的方法, 通过控制方程, 利用边界条件(10)中的式 1, 式 3 以及竖向连续条件(7)~(9)可获得该系统的正交关系:

$$\sum_{i=1}^n (n^2 - 1) m_{vi} \int_{H_{i-1}}^{H_i} Z_{im} Z_{im'} dz + \sum_{j=1}^N n^2 m_{v\perp j} \int_{H_{\perp(j-1)}}^{H_{\perp j}} Z_{\perp jm} Z_{\perp jm'} dz = 0 \quad (22)$$

式中 $H_{\perp 0}=H_n$ 。根据该正交关系以及初始条件, 可求得 A_m :

$$\begin{aligned} A_m &= \frac{(n^2 - 1) E_m + n^2 F_m}{(n^2 - 1) G_m + n^2 H_m}, \quad E_m = q_0 \sum_{i=1}^n m_{vi} \int_{H_{i-1}}^{H_i} g_{im} dz; \\ F_m &= q_0 \sum_{j=1}^N m_{v\perp j} \int_{H_{\perp(j-1)}}^{H_{\perp j}} g_{\perp jm} dz; \quad G_m = \sum_{i=1}^n m_{vi} \int_{H_{i-1}}^{H_i} g_{im}^2 dz; \\ H_m &= \sum_{j=1}^N m_{v\perp j} \int_{H_{\perp(j-1)}}^{H_{\perp j}} g_{\perp jm}^2 dz \end{aligned} \quad (23)$$

至此, 式 (12)~(14) 中各项待定参数均已确定, 代入后即可获得各孔压值。通过孔压消散情况, 可求得任一深度土体平均固结度和各土层平均固结度以及整个系统的平均固结度。

竖井打设区各土层平均固结度为

$$\bar{U}_i = 1 - \sum_{m=0}^{\infty} A_m e^{-\beta_m t} \int_{H_{i-1}}^{H_i} g_{im}(z) dz / (h_i q_0) \quad (24)$$

下卧层各土层平均固结度为

$$\bar{U}_{\perp j} = 1 - \frac{1}{h_{\perp j} q_0} \sum_{m=0}^{\infty} A_m e^{-\beta_m t} \int_{H_{\perp(j-1)}}^{H_{\perp j}} g_{\perp jm}(z) dz \quad (25)$$

整个土层按孔压定义的平均固结度为

$$\bar{U}_p = \left(\sum_{i=1}^n h_i \bar{U}_i + \sum_{j=1}^N h_{\perp j} \bar{U}_{\perp j} \right) / H_t \quad (26)$$

整个土层按变形定义的平均固结度为

$$\bar{U}_s = \frac{\sum_{i=1}^n h_i m_{vi} \bar{U}_i + \sum_{j=1}^N h_{\perp j} m_{v\perp j} \bar{U}_{\perp j}}{\sum_{i=1}^n h_i m_{vi} + \sum_{j=1}^N h_{\perp j} m_{v\perp j}} \quad (27)$$

4 算例分析

为了便于实际应用, 按照上述计算原理, 已将本文解编制成应用程序。用该程序对一算例进行了分析。该算例的情况为: $r_w=0.035$ m, $r_e=0.63$ m, $s=2$; 竖井渗透系数 $k_w=2 \times 10^{-5}$ m/s, 打设深度为 20 m, 各土层所用计算参数如表 1 所示。

表 1 土层参数

Table 1 Geotechnical data of the layered system

土层 编号	k_v / $10^{-9} \text{ m} \cdot \text{s}^{-1}$	K_h / $10^{-9} \text{ m} \cdot \text{s}^{-1}$	K_s / $10^{-9} \text{ m} \cdot \text{s}^{-1}$	m_v / MPa^{-1}	h /m
$i=1$	1	2	1	0.133 4	5
$i=2$	2	4	2	0.667 0	15
$j=1$	2	4	2	0.333 5	10

为便于研究, 取 $T_c = C_{hi}t/d_c^2$ 作为参考时间因数, 以 T_c 为横坐标, 孔压平均固结度为纵坐标。图 2 反映了按孔压定义的系统平均固结度与按变形定义的系统平均固结度比较情况。显示按孔压定义的平均固结度与按孔压定义的平均固结度二者并不一致, 成交叉发展。式 (26) 和式 (27) 也说明了这一点, 除了各土层的体积压缩系数完全相同, 才能保证任一时刻两种定义的平均固结度相一致。固结初始阶段, 按变形定义的平均固结度大于按孔压定义的平均固结度, 而后期二者则较为接近。这主要因为前期固结以竖井加固区两土层为主, 而固结前期两者平均固结度相差较大, 后期则较为接近, 如图 2 所示。图 3 反映了各层固结情况。

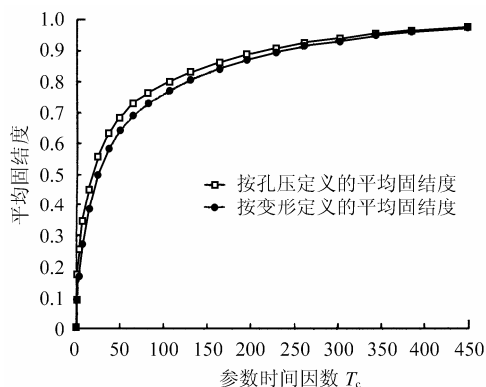


图 2 按孔压定义和按变形定义的平均固结度比较

Fig. 2 Comparison of average degree between defined by pore pressure and defined by deformation

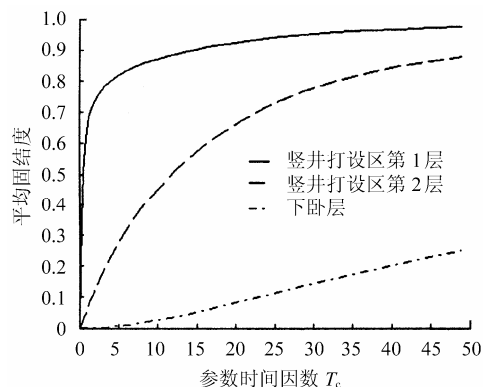


图 3 各土层平均固结度

Fig. 3 Average consolidation degree of each layer

若地基表面无硬表层, 即第 1 层土体的体积压缩系数与第 2 层土体的体积压缩系数相同, 图 4 反映了有无硬表层时各层平均固结度对比情况。从图中可以看出地表存在硬表层可以加速各土层固结。

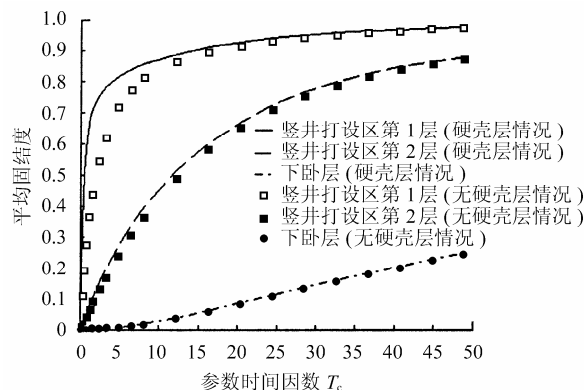


图 4 有无硬表层时各土层平均固结度

Fig. 4 Average consolidation degrees of each layer with and without crust

5 结 语

实际工程中, 竖井加固软土地基通常具有成层性, 竖井未能打穿软土层的情况也时有发生。本文在现有竖井地基固结理论基础上, 对成层未打穿竖井地基情况进行了研究, 获得了该系统的正交关系, 同时给出了具体求解过程及其解答。本文解答具有广泛的适用性, 若竖井打设区和下卧层层数均为一层, 即可获得现有简单未打穿竖井地基的固结解答。本文解答表明, 整个土层按孔压定义的平均固结度与按变形定义的平均固结度一般并不相同, 只有在所有土层体积压缩系数相同时, 其结果才会相同。硬表层的存在会加速其下土层的固结。

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